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Prediction of Structure-Borne Sound in Buildings Using the Substructure Synthesis Method

Kenichi Takebayashi, Aya Tanaka and Kei Andow[1)]

ABSTRACT

Accurate predictions of structure-borne sound in buildings using numerical simulation method, such as the finite element method, require detailed modeling, which consumes large amounts of memory and computation time. The substructure synthesis method has been progressively developed since 1970 to obtain dynamic characteristics of large-scale structures, such as machines and vehicles. This method determines the dynamic characteristics of a structure by synthesizing the vibration modes in each substructure, which can be computed independently. However, it is time consuming when applied to analyze a building due to the computational load. This study presents two approaches to reduce the computation time of the substructure synthesis method. The first approach is to reuse the calculation results. In buildings, once a wall, column, or beam has been analyzed, the results can be applied to the other walls, columns, and beams having the same material and shape. The second approach is to multiplex the algorithm. A group of substructures are analyzed and synthesized into a new substructure. This procedure is repeated until the substructures become the entire structure. In this study, these two approaches are applied to analyze a frame structure and are shown to be effective in reducing the computation time without decreasing the calculation accuracy.

Contents

I. Introduction

Prediction of structure-borne sound using the finite element method (FEM) with detailed modeling, consumes large amount of memory and computation time. Although the supercomputers have become more advanced and enable large-scale problem solving, operating them is still extremely expensive. Furthermore, design changes are made frequently in design phase, so it is not practical to use supercomputers. Thus, energy-based simulations, such as the statistical energy analysis (SEA), are used[1)]. These simulations expect lower computational costs than the FEM. But in a low-frequency range, accurate prediction is difficult because of the low modal density. The substructure synthesis method has been progressively developed since 1970 to obtain dynamic characteristics of large-scale structures, such as machines and vehicles[2)]. This method can independently analyze substructures and can synthesize them to obtain the dynamic characteristics of the entire structure. Therefore, this method can analyze if the target structure is too large to analyze at once. However, computation time and storage/memory needed to store computed results for each substructure would also increase along with the increasing of the number of substructures.

In this study, we investigated the efficiency of the substructure synthesis method in terms of the computational cost, with a focus on computation time. To this end, two bottlenecks of the algorithm are presented. Then, two approaches for reducing computation time are investigated. The first approach is to reuse the calculation results and the other is to multiplex the algorithm.

1） Andow Environmental Consultant

Keywords：Structure borne sound, Numerical simulation, Modal analysis, Substructure synthesis, Eigenvalue problem,

II. Substructure Synthesis Method

1. Modal Analysis and Eigenvalue Problem

Substructure synthesis method is based on the modal analysis theory. Modal analysis is used to obtain the dynamic characteristics of a structure/space using a superposition of eigenmodes with initial condition described in Eq. (1).

$$x_q(\omega) = \sum_{m=1}^{M} \frac{\phi_{mq}\phi_{mp}}{\Omega_m - \omega^2} f_p(\omega) \tag{1}$$

where, x_q is the dynamic characteristics at the receiver point q, ω is an angular frequency, Ω_m is the mth order eigenvalue, ϕ_{mq} and ϕ_{mp} are the mth eigenmode at the receiver point q and the driving point p, f_p is the driving force at the point p. Modal analysis can immediately calculate the dynamic characteristics at an another point if the eingenmodes have already been computed. Compared with the approach of solving system equation at each frequency, the approach mentioned above has a much shorter computation time.

Eigenmodes and eigenvalues are obtained by solving the generalized eigenvalue problem $(\mathbf{K}\text{-}\omega^2\mathbf{M})\{x\} = \{0\}$. **K** and **M** are the stiffness and mass matrix computed by the FEM, respectively. This eigenvalue problem can be solved in various ways, such as the subspace[3] or Lanczos[4] (Arnoldi[5]) method or substructure synthesis method. In particular, the substructure synthesis method can obtain the eigenmodes of the entire structure/space by synthesizing the eigenmodes of each substructure, which are computed independently. Therefore, this method is applicable when a target structure/space is too large to solve the eigenvalue problem at once.

2. Algorithm of the Substructure Synthesis Method

In this study, component mode synthesis[6] (CMS), one of the substructure synthesis algorithms is used. The CMS algorithm is described below. A structure divided into two substructures is shown in Fig.1. Nodes of each substructure are classified into two areas: internal and coupling area. The internal area does not have the common nodes to the other substructure. Conversely, the coupling area comprises only common nodes. The **K** matrix of each substructure is represented in equation (2) and (3), where a and c represent the internal areas of each substructure, b represents the coupling area of both substructures.

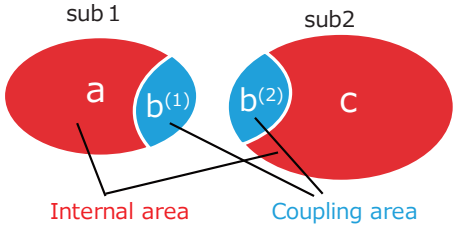

Fig.1 Schematic of the Two Substructures. Each Substructure Comprises Internal (red) and Coupling Area (blue)

$$K_1 = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb}^{(1)} \end{bmatrix} \tag{2}$$

$$K_2 = \begin{bmatrix} K_{bb}^{(2)} & K_{bc} \\ K_{cb} & K_{cc} \end{bmatrix} \tag{3}$$

The generalized eigenvalue problems on each internal area (shown in Eq. (4) and (5)) are solved using the Lanczos method. As a result, eigenmode matrices ϕ_a and ϕ_c and eigenvalues λ_a and λ_c are obtained. Eigenmode matrices ϕ_a and ϕ_c comprise 1st to Mth eigenvectors in ascending order. Generally, the order M is set much smaller than the degree of freedom of the internal area.

$$(K_{aa} - \omega^2 M_{aa})\{x_a\} = \{0\} \tag{4}$$

$$(K_{cc} - \omega^2 M_{cc})\{x_c\} = \{0\} \tag{5}$$

Each internal area are degenerated into coupling area using the Guyan reduction[7].

$$\overline{K}_{bb}^{(1)} = \begin{bmatrix} T_{ab}^{\ t} \\ I \end{bmatrix}^t \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \begin{bmatrix} T_{ab} \\ I \end{bmatrix}, \tag{6}$$

$$\overline{K}_{bb}^{(2)} = \begin{bmatrix} I \\ T_{cb}^{\ t} \end{bmatrix}^t \begin{bmatrix} K_{bb} & K_{bc} \\ K_{cb} & K_{cc} \end{bmatrix} \begin{bmatrix} I \\ T_{cb} \end{bmatrix} \tag{7}$$

where $\overline{K}_{bb}^{(1)}$ and $\overline{K}_{bb}^{(2)}$ are the degenerated matrices and T_{ab} and T_{cb} are the transformation matrices computed using Eqs. (8) and (9) respectively.

$$T_{ab} = -K_{aa}^{-1} K_{ab} \tag{8}$$

$$T_{cb} = -K_{cc}^{-1} K_{cb} \tag{9}$$

The degenerated matrix for the coupling area is assembled and analyzed to obtain eigenmodes matrix ϕ_b and eigenvalue λ_b.

$$\{(\overline{K}_{bb}^{(1)} + \overline{K}_{bb}^{(2)}) - \omega^2(\overline{M}_{bb}^{(1)} + \overline{M}_{bb}^{(2)})\}\{x_b\} = \{0\} \tag{10}$$

The equation of motion for all the areas in physical coordinate system is transformed into the modal coordinate system by the transforming matrix T_{all}. The degree of freedom of Eq. (11) is $M_{all}=M_b+M_a+M_c$, where M_b, M_a and M_c are the highest order of coupling area *b*, internal area *a* and *c*, respectively. M_{all} is much smaller than the degree of freedom of the equation of motion in physical coordinate system. Solving Eq. (11) provides the eigenmodes matrix ξ in modal coordinate system and eigenvalue λ_{all}. Equation 11 is given as follows:

$$[T_{all}]^t\left(K_{all}-\omega^2 M_{all}\right)[T_{all}]\{\xi\}=\{0\}, \tag{11}$$

where T_{all} , K_{all} and M_{all} can be written as

$$[T_{all}]=\begin{bmatrix} T_{ab}\phi_b & \phi_a & 0 \\ \phi_b & 0 & 0 \\ T_{cb}\phi_b & 0 & \phi_c \end{bmatrix}, \tag{12}$$

$$[K_{all}]=\begin{bmatrix} K_{aa} & K_{ab} & 0 \\ K_{ba} & K_{bb}^{(1)}+K_{bb}^{(2)} & K_{bc} \\ 0 & K_{cb} & K_{cc} \end{bmatrix}, \tag{13}$$

$$[M_{all}]=\begin{bmatrix} M_{aa} & M_{ab} & 0 \\ M_{ba} & M_{bb}^{(1)}+M_{bb}^{(2)} & M_{bc} \\ 0 & M_{cb} & M_{cc} \end{bmatrix}, \tag{14}$$

Finally, eigenmodes of entire structure ϕ_{all} on physical coordinate are obtained by re-transforming (Eq. 15).

$$\phi_{all}=T_{all}\xi \tag{15}$$

3. Application of the Substructure Synthesis Method to a Frame Structure

The substructure synthesis method described above was implemented to obtain dynamic characteristics of the frame structure shown in Fig. 2. This frame structure was a 1/20 - scaled model of a ten-story building and comprised PVC beams and columns with 3 cm x 3 cm cross section. The height of the frame structure was 1.5 m and its width was 0.33 m. The material properties of the frame structure are shown in Table 1. The bottom of the columns was fixed. First, the analytical model with a finite element (FE) mesh is created. The frame structure was discretized by 5 mm, 8 - node rectangular elements. The number of nodes was 162,876 (488,626 dofs). Moreover, 500 eigenmodes were obtained using the Lanczos and substructure synthesis

Table 1 Material properties of PVC

Young's modules ,GPa	Poisson's ratio	Density, kg/m^3	Modal damping
2.843	0.25	1385	0.02

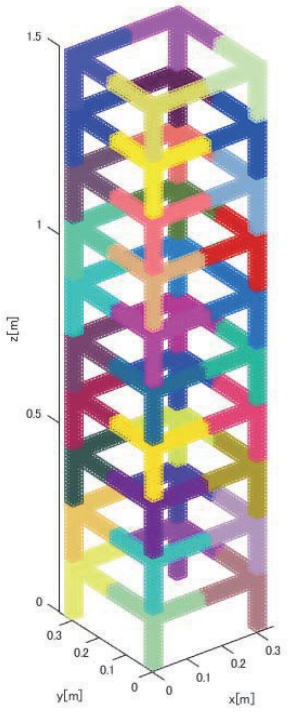

Fig.2 A 1/20-Scale 10-Story Frame Structure Model Divided into 40 Substructures. Each Substructure is Indicated with a Different Color

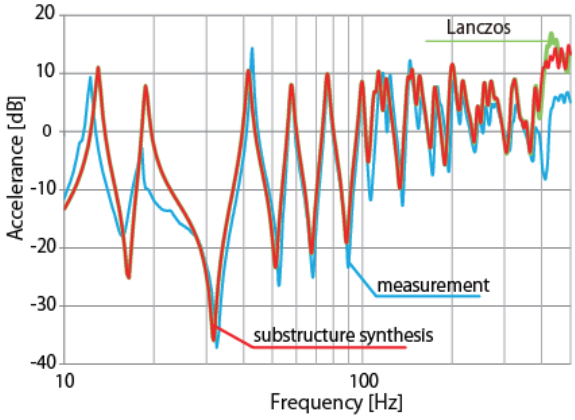

Fig.3 Measured and Calculated Accelerances at the Top of the Column

methods. In case of the substructure synthesis method, the FE model was divided into 40 substructures and was synthesized according to the calculation procedure expressed above. All computations (the Lanczos and the substructure synthesis methods) were performed in parallel using a personal computer with eight physical cores (CPU: Intel Xeon E5-2630 v3 2.4 GHz, OS: Windows 7 64bit, RAM: 32 GB).

Fig. 3 shows the measured and calculated results of the transfer function (driving-point accelerance, lateral-direction excitation) at the top of the column. All the results agree well with the measured values. The discrepancy above 400 Hz was attributed to modal damping. In this case, modal damping was assigned a constant value (0.02), but modal damping in the high frequency region could be higher than 0.02. This value will be estimated accurately in future work.

Fig. 4 shows the computation time for the frame structure. The Lanczos method took 1640 s, whereas, the substructure synthesis method took 434 s, achieving a 73 % reduction in computation time.

5. Bottlenecs in the calculation

Fig. 5 shows the breakdown of the computation time for the frame structure model using the substructure synthesis method.

As observed, the CMS algorithm has two bottlenecks.

The first bottleneck is computation of the transformation matrix in Eqs. (8) and (9). These equations include inverse matrix K_{aa}^{-1} and K_{cc}^{-1} which require lower-upper (LU) or Cholesky factorizations. These equations also consume large amounts of memory, because the inverse matrices lose their sparsity pattern and become full matrices.

The second bottleneck is to solve the eigenvalue problem for coupling area given in Eq. (10). Increasing the number of substructures in the target model will also increase the number of coupling area and, consequently, the computation time.

III. Reuse and Multiplexing

We applied two approaches to reduce the computation time at the bottlenecks: reusing of the calculation results and multiplexing the algorithm.

1. Reusing

In a building, many structural elements, such as beams, columns and walls, have the same material composition and size. Reusing the calculation results may prove to be an efficient way to reduce the computation time and consumption of large amounts of memory, especially for buildings, because, once a wall, column, or beam has been analyzed, the result can be applied to the other walls, columns, and beams.

In this study, using a reuse procedure, we attempted to reduce computation time and memory consumption for solving frame structure model described above. Only two substructures, a bottom and a top substructure with different boundary conditions, were analyzed, and the calculation results (the eingenmode matrix of internal area, transformation matrix, and degenerated matrix) were stored. Fig. 6 shows the substructures to be analyzed. In order to apply the calculation results to other substructures, the computed results must be rotated and permuted for attaining the same nodal order between the original and the copied substructures. Although, these procedures require additional computation, the computation time for rotation and permutation is much shorter than the time consumed at the first bottleneck without reuse procedure. With regard to memory consumption, reuse procedure requires to store matrices of original substructures shown in Fig. 6. It is only 1/20 use of memory compared to without reuse procedure.

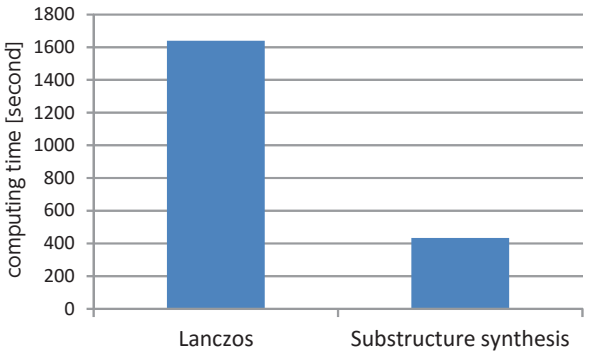

Fig.4 Computation Time for Obtaining 500 Eigenmodes of the Frame Structure Model Using the Lanczos and Substructure Synthesis Method

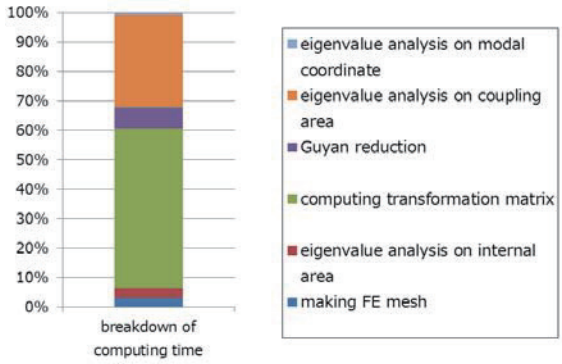

Fig.5 Breakdown of the Computation Time for a Frame Structure Model Using the Substructure Synthesis Method

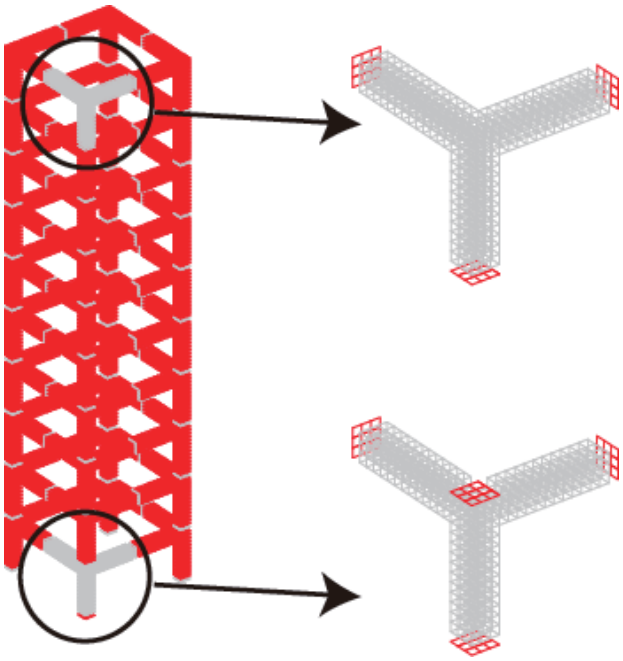

Fig.6 Substructures to be Analyzed. Bottom and Top Substructures Have Different Boundary Condition; Internal Area (gray line), Coupling Area (red line)

Fig. 7 shows the 1st to 30th order eigenfrequencies computed with/without the reuse procedure. These values are in good agreement regardless of whether the reuse procedure is employed. Table 2 shows the computation time with/without the reuse procedure. As observed, the computation time with the reuse procedure is approximately 40 % shorter than that without the reuse procedure.

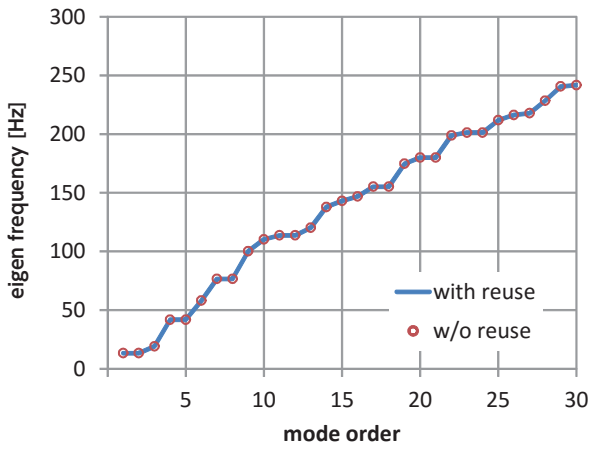

Fig.7 Eigenfrequency Computed with/without the Reuse Procedure

2. Multiplexing

The second bottleneck is to solve eigenvalue problem on the coupling area. We attempted to multiplex the algorithm in order to reduce the number of nodes in coupling area.

To this end, a two-stage algorithm was adopted. A small group of substructures was analyzed and synthesized into a new substructure. Then this new substructure was synthesized again with the other synthesized substructures (Fig. 8). In case of the frame structure, 40 substructures were classified into eight groups; each group had four to six substructures. The number of coupling area was 76, and each coupling area had 49 nodes. The total number of nodes in the coupling area was 3,724. Using the two-stage algorithm, the number of nodes in the coupling area was reduced to 196-294 in the first stage in each small group, and to 1,470 in the second stage.

The computation time for the eigenvalue analysis of the coupling area is 3 s in the first stage and 48 s in the second stage; without multiplexing, the computation time was 132 s. Although the multiplex approach reduced the computation time, this reduction was only 34 s (434-400 s.) because the multiplex algorithm requires another computations (updating the transformation matrix, solving the eigenvalue problem in the modal coordinate system, and re-transforming the eigenmodes from the modal coordinate system to the physical coordinate system).

3. Combination of the Reuse and Multiplex Strategies

When using both the reuse and multiplex strategies, the computation time is 217 s. Thus, the computation time when both strategies are used is half of that when CMS without reuse or multiplexing is used, and 1/8 of that when the Lanczos method is used.

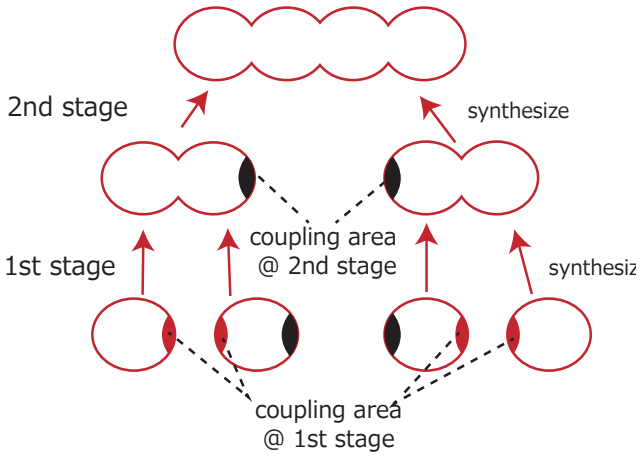

Fig.8 Schematic of the Two-Stage Multiplex Algorithm

Table 2 Computation Time Using Different Simulation Strategies

Lanczos	CMS with			
	-	multiplex	reuse	multiplex, reuse
1640 s	434 s	400 s	251 s	217 s

IV. Conclusions

In this study, the substructure synthesis method was used to obtain dynamic characteristics of the frame structure model. This approach effectively reduced the computation time compared with the Lanczos method. However, two bottlenecks were identified in the substructure synthesis algorithm: computing the transformation matrix and conducting the eigenvalue analysis for the coupling area. To eliminate these bottlenecks, we employed two strategies. The first strategy was the reuse of the computation results. Specifically, the calculation results obtained for one substructure were applied to other substructures with the same shapes, properties and boundary conditions via matrix rotation and permutation. This procedure reduced the computation time and memory consumption. The second strategy was to multiplex the algorithm using the two-stage algorithm. Although, the computation time for the eigenvalue analysis of the coupling area was much shorter than that without multiplexing, the reduction in overall computation time was rather small because of the need for additional calculations. The combination of reuse and multiplexing was very effective in reducing the computation time.

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